ALE AND LAGRANGIAN FINITE ELEMENT METHODS FOR FREE SURFACE FLOWS

E. Oñate[a], **J. García**[a] and **S.R. Idelsohn**[b]

[a]International Center for Numerical Methods in Engineering (CIMNE)
Universidad Politécnica de Cataluña
Gran Capitán, s/n. 08034 Barcelona. Spain
onate@cimne.upc.es

[b]International Center for Computational Methods in Engineering (CIMEC)
INTEC (CONICET – Universidad Nacional del Litoral)
Güemes 3450, 3000 Santa Fe, Argentina
Sergio@ceride.gov.ar

The accurate analysis of free surface flows is of considerable interest in many areas of engineering. Typical examples are ship hydrodynamics and open channel flows and cascade flows in hydraulic and environmental engineering, among many others. The computational challenges are mainly due to the difficulty of solving numerically the incompressible Navier-Stokes equations coupled with the constraint equation stating that at the free surface boundary the fluid particles remain in that surface, which position is unknown.

The paper presents a stabilized finite element method which allows to overcome the above mentioned problems. The starting point are the modified governing differential equations for an incompressible viscous flow with the free surface condition incorporating the necessary stabilization terms via a finite calculus (FIC) procedure [1]. This technique is based on writing the different balance equations over a domain of finite size and retaining higher order terms. These terms incorporate the ingredients for the necessary stabilization of any transient and steady state numerical solution. In addition, the modified differential equations can be used to derive an iterative numerical scheme for computing the stabilization parameters.

The FIC differential equations for the arbitrary lagrangian-eulerian (ALE) description are first solved in time using a semi-implicit fractional step approach and the standard Galerkin finite element formulation. This leads to a stabilized system of discretized equations allowing for equal order linear interpolations of the velocity and pressure variables over the elements. Unstructured grids of linear tetrahedra elements are used. Free surface wave boundary effects are accounted for either by moving the free surface nodes in a lagrangean manner, or else for via the introduction of a prescribed pressure at the free surface computed from the wave height.

In second part of the paper the stabilized ALE description is particularized to the full lagrangian form. This allows to treat free surface flows involving large distortions of the fluid surface [2]. Details of the lagrangian formulation including the procedure for updating the mesh in a fast and efficient manner are given.

Examples of application of the ALE and lagrangian formulations to a variety of ship hydrodynamics and other free surface flow situations are presented.

References

[1] E. Oñate and J. García, "A finite element method for fluid-structure interaction with surface waves using a finite calculus formulation". *Comp. Meth. Appl. Mech. Engng.*, v. 191 6-7, p. 635 – 660, 2001.

[2] S.R. Idelsohn, E. Oñate and F. del Pin, "A lagrangian meshless FEM applied to fluid-structure interaction problems". In press in *Computers and Structures*, 2003.